Determining, From Experimental Data, the Exponent of the Divergence Function for Refracted Waves

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DETERMINING THE EXPONENT OF THE DIVERGENCE FUNCTION FOR REFRACTED WAVES FROM EXPERIMENTAL DATA

I. S. Berzon

A method was suggested for determining the exponent n of the divergence function for refracted waves according to the curve of the relation of amplitude to distance. The curve was obtained through observations with respect to the correlation method of refracted waves.

The problem has been investigated concerning the adaptability of this method to the case when the exponent n and the coefficient of absorption α in a refracting layer are varying in magnitude.

It was noted that the accuracy of determining the coefficient of absorption in a refracting layer essentially depends upon the exactness of the determination of the exponent n of the divergence function.

Examples of determining the index n according to experimental data were introduced.

If a wave, propagating in a medium with less velocity, drops at the critical angle onto the boundary of a layer with greater velocity, then in the medium with greater velocity along the boundary of separation there is propagated a wave which "pulls" in its wake (in the medium with less velocity) a refracting (head) wave [1, 2]. The recording of these refracting (head) waves is based on the correlation method of refracted waves. In the future we shall briefly designate these waves as refracted in accordance with terminology which is acceptable in seismic research.

The multiple experimental works which were conducted according to the correlation method of refracted waves [3 - 7], show that refracted waves fade with distance, whereupon the degree of fading out [damping] depends on the sismological construction of the medium -- on the capacity of the layer with which a formation of these waves is connected, on the stratification depth and angle of slope of the separation boundary, on the absorption characteristics of the refracting layer and the overlying medium, etc.

The factors which cause the decrease in amplitude of refracted waves in relation to distance can schematically be separated into two groups.

One set of conditions is related to the fact that according to measurement of discharge from the epicentre there is an increase in the surface of the sliding wave front, propagating in the medium with greater velocity, i.e. a "divergence" takes place of the sliding front at the expense of which the density of wave energy is reduced. This subsequently involves a decrease in energy density for refracted waves, insofar as the known conditions of intensity regularity and coalescence on the separation boundary must be observed. For an increase in distance from the epicentre there is also an increase in the surface of the front of the refracted wave itself (divergence of the front). The divergence of the fronts of sliding and refracted waves depends on the velocity of the characteristic medium, on the ~~depth, and on the~~ form of stratification and the magnitude of the refracting layer.

Another set of conditions causing attenuation of refracted waves with respect to distance, is tied up with the absorbing properties and heterogeneity of the medium, and also with the energy output in the superincumbent medium which generates refracted waves.

Subsequently we shall use the term "absorption", understanding by this the losses not only at the expense of non-ideal resilience of the medium and emanations in the superincumbent medium, but also at the expense of heterogeneity of the refracting layer, in particular, the losses at the expense of frequent reverberations in the case of thin vertical lamination (stratification) also at the expense of emanations in the underlying medium, in the case of thin horizontal laminations. In such a way one may present the amplitude of a refracted wave in the following form

$$A = \frac{C}{f(x)} e^{-(\alpha_2 r_2 + \alpha_1 r_1)}$$

where C is a constant; x is the distance from the epicentre, greater than the abscissa x_{init}, the initial point of the refracted wave; α_1 and α_2 are constants -- coefficients of absorption in the refracting layer and in the covering medium, respectively; r_2 and r_1 are the lengths of the path of sliding and refracted waves, respectively.

In the formula stated, $e^{-(\alpha_2 r_2 + \alpha_1 r_1)}$ is the term which determines the amplitude decrease of refracted waves at the expense of absorption, f(x) is the function which determines the amplitude decrease of refracted waves at the expense of the divergence of sliding and refracted wave fronts. In the future we shall term these divergence functions for refracted waves ~~briefly as divergence~~ functions.

The problem of the aspect of the divergence function f(x), as is pointed out further, is little investigated at present and its investigation involves severe difficulties. Together with this, the problem offers great interest in the investigation of the physics of of refracted wave propagation, as well as in the determination of absorption characteristics of different media.

In actual work a method is suggested for determining the divergence function for refracted waves in line with experimental data, obtained as a result of works in the correlation method of refracted waves, and examples of the practical application of the method are cited.

1. A Brief Summary of the Theoretical Work in Reference to the Investigation of Divergence Functions for Refracted Waves

The problem of variation with amplitude range of refracted waves in the case of a horizontal separation boundary of two non-absorbing media or, in other words, the problem of determining the divergence functions for refracted waves was investigated in a series of works by different authors. Jeffreys [8] investigated the problem for the case of wave incidence having the form of a single impulse on the separation boundary of two non-absorbing liquid media and showed that the amplitude of sound potential of refracted waves over great distances from the source varied according to the ratio

$$A = \frac{C}{x^2} \qquad (1)$$

where C is a constant.

Consequently, the divergence function f(x) is an exponential function.

Such a relation was suggested by Muskat (9) and several other authors who are investigating the dynamic problem of elastic propagation of waves in a solid medium with a separation boundary.

In detail, amplitude variation of refracted waves with distance is investigated in a series of works by Brekhovskikh [10-12] for the case when the principal point of fixed sinusoidal oscillations is established in a liquid medium with separation boundary velocities. The formula of Brekhovskikh for amplitude of sound potential of refracted waves for the cases of non-absorbing media has the following form [11]:

$$A = \frac{p}{mR\left(\frac{kR}{2}\right)^{\frac{1}{4}}} \frac{|F(\eta)|}{\eta^{3/2}} \frac{1}{\left(\sin\chi_0 \cdot \cos\chi \ \cos^3 \frac{\chi_0 - \chi}{2}\right)^{\frac{1}{2}}} \qquad (2)$$

where $p = \frac{V_1}{V_2}$ is the ratio of velocity in the covering medium and in the refracting layer, $m = \rho_2 / \rho_1$ is the ratio of density in the refracting layer and in the covering medium, $k = 2\pi/\lambda_1$ is the length of the wave in the covering medium, $\eta = \sqrt{2kR}\cdot\sin\frac{\chi_0 - \chi}{2}$.
The angles χ_0 and χ and distance R are shown in Figure 1.

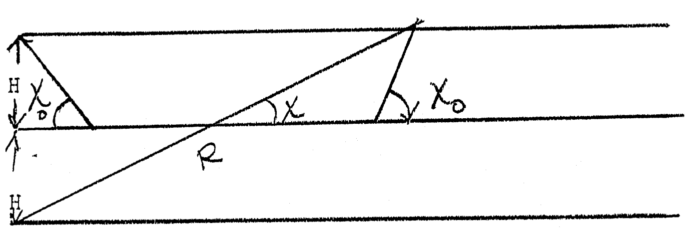

Figure 1. Determining the divergence function for a refracted wave (according to L. M. Brekhovskikh)

The function F(η), the modulus of which enters into formula (2), is presented in respect to the series of exponents $1/\eta^2$ and η^2. Formula (2) offers great interest because after some transformation it lends itself to observance of the ratio of refracted wave amplitude to distance for any values of x -- in this number and in the neighborhood of the origin. In work (2) an investigation of this ratio for the case of great distances $x \gg 2H$ is introduced, where the ratio assumes the form of (1); it is known that the amplitude A decreases in an inverse proportion to the square of the distance. In the following paragraph we shall discontinue the detailed examination of formula (2).

The most complete investigation of dynamic properties of refracted waves is given in the work of N. V. Zvolinskiy and L. P. Zaytsev [3]. In this work the dynamic problem of reflection and refraction waves on a horizontal boundary of two elastic non-absorbing fluid media is solved. The problem is solved for the dimetric case, by which the source which stimulates the single impulse is investigated. As a result of the investigation it is shown that the coalescent amplitude for refracted waves decreases according to the principle:

$$A = \frac{C}{x^{3/3}} \qquad \text{[sic]} \qquad [3]$$

where x is the horizontal distance from the epicentre to the point of observation, and C is a constant.

Thus, from the investigation of all enumerated works it follows that in the case of horizontal separation boundaries the amplitude of refracted waves varies according to the principle of the exponential function

$$A = \frac{C}{x^n} \qquad (4)$$

where x^n is the divergence function, and n is the exponent of divergence function or, in short, the index of divergence, numerically equal to three-halves or two.

2. The Determination of the Relation of Index n to Distance According to L. M. Brekhovskikh's Formula

As was pointed out in the preceding paragraph, in the work of Brekhovskikh [12] it was shown that for great distances x 2H, the amplitude of refracted waves, which is determined from formula (2), decreases with distance according to the principle $1/x^2$. We are arrested now in the investigation of the relation of amplitude A to distance by the minimum practicable distance x, i.e. in the neighborhood of the initial point of the refracted wave hodograph. Linked to this is the fact that formula (2) is of unwieldy size and without additional calculation according to it, it is difficult to maintain the ratio of amplitude to distance x, and for a more descriptive presentation of this ratio we shall proceed with the following method. At each point which is at a distance x from the origin, we select the exponential function

$$\bar{A} = \frac{C}{x^n} \qquad (5)$$

so that the derivative $\frac{d \ln \bar{A}}{d \ln x}$ at this point which characterizes a variation of amplitude with distance from its neighborhood, is equal to the derivative $\frac{d \ln A}{d \ln x}$ from the function $A = A(x)$, determined from formula (2). Accordingly, the index n in the general case may be different for equal points, i.e. n can appear as a function of distance x.

For the exponential function (5) appearing in a two-dimensional coordinate system

$$y = \ln x \qquad\qquad z = \ln \bar{A},$$

then curve (5) is transformed directly to

$$z = -ny + C' \qquad (6)$$

where C is a constant.

The angle coefficient dz/dy direct from (6) is equal to n. Now let us introduce into this coordinate system curve (2) and we shall define the angle coefficient $n(x_{init})$ as the tangent to this curve at the origin. Expressing in formula (2) the magnitudes and R by x and H, and angle by the relation of velocity p, we obtain the following expression for index n:

$$n = -\frac{dz}{dy} = -\frac{d\left[\ln \frac{F(\eta)}{\eta^{7/3}}\right]}{d(\ln x)} +$$

$$+ \frac{1}{2}\frac{d}{d(\ln x)} \ln\left\{x\sqrt{1 - p^2}\left[\sqrt{x^2 + 4H^2} + px + 2H\sqrt{1 - p^2}\right]^{3/2}\right\} \qquad (7)$$

where

$$\eta = 2\sqrt{\pi\frac{H}{\lambda}}\sqrt{\sqrt{\left(\frac{x}{2H}\right)^2 + 1} - p\frac{x}{2H} - \sqrt{1 - p^2}} \qquad (8)$$

Let us introduce the designation

$$\frac{1}{2}\frac{d}{d(\ln x)} \ln\left\{x\sqrt{1 - p^2}\left[\sqrt{x^2 + 4H^2} + px + 2H\sqrt{1 - p^2}\right]^{3/2}\right\} = n_1(x), \qquad (9)$$

$$\frac{-d\left[\ln \frac{|F(\eta)|}{\eta^{3/2}}\right]}{d(\ln x)} = n_2(x) \tag{10}$$

Performing the differentiation on the left side of formula (9), we derive the following expression for $n_1(x)$:

$$n_1(x) = \frac{1}{2} + \frac{3}{4}\,\frac{p + \dfrac{\frac{x}{2H}}{\sqrt{\left(\frac{x}{2H}\right)^2 + 1}}}{p\,\frac{x}{2H} + \sqrt{\left(\frac{x}{2H}\right)^2 + 1} + \sqrt{1 - p^2}}\;\frac{x}{2H} \tag{11}$$

At the origin of the refracted wave hodograph, i.e. for

$$\frac{x}{2H} = \frac{p}{\sqrt{1 - p^2}} \tag{12}$$

$$n_1(x_{init}) = 0.5 + 0.75\,p^2. \tag{13}$$

The function $n_2(x)$ after several transformations can be written in the following form

$$n_2(x) = -\frac{1}{2}\,\frac{x}{2H}\,\frac{\dfrac{\frac{x}{2H}}{\sqrt{\left(\frac{x}{2H}\right)^2 + 1}} - p}{\sqrt{\left(\frac{x}{2H}\right)^2 + 1} - n\frac{x}{2H} - \sqrt{1 - p^2}}\;\frac{d\left[\ln \frac{|F(\eta)|}{\eta^{3/2}}\right]}{d \ln \eta} \tag{14}$$

For computing the derivative

$$\frac{d \ln \frac{|F(\eta)|}{\eta^{3/2}}}{d \ln \eta},$$

let us separate the function $\frac{F(\eta)}{\eta^{3/2}}$ according to degrees of , introduced in work [11]:

$$\frac{F(\eta)}{\eta^{3/2}} = \sqrt{\pi}\, e^{3\pi i/8} \left\{ \frac{4}{G(\frac{1}{4})} \left[1 + \frac{3\eta^2}{2} i - \frac{3\cdot 7}{2!\,1\cdot 3}\left(\frac{\eta^2}{2}\right)^2 - \frac{3\cdot 7\cdot 11}{3!\,1\cdot 3\cdot 5}\left(\frac{\eta^2}{2}\right)^3 + \ldots \right] - \right.$$

$$\left. - \frac{\sqrt{2}\,(1+i)}{G(3/4)}\, \eta \left[1 + \frac{5}{3}\left(\frac{\eta^2}{2}\right) i - \frac{5\cdot 9}{2!\,3\cdot 5}\left(\frac{\eta^2}{2}\right)^2 - \frac{5\cdot 9\cdot 13}{3!\,3\cdot 5\cdot 7}\left(\frac{\eta^2}{2}\right)^3 i + \ldots \right] \right\}. \quad (15)$$

Deriving from (15) the modulus of the function F( ) and deriving the differential function $\ln \left| \frac{F(\eta)}{\eta^{3/2}} \right|$, with respect to ln , we obtain the relationship $n_2 = n_2(\eta)$ in accordance with degrees of η^2. The origin $\eta = 0$ and $n_2(x_{init})$ is expressed in the following simple formula:

$$n_2(x_{init}) = \sqrt{2\pi \frac{H}{\lambda}}\, \frac{\sqrt{2}}{4}\, p(1 - p^2)^{\frac{1}{4}} \;\; \frac{G\,\frac{1}{4}}{G\,\frac{3}{4}} \;\; = 0.871 p(1 - p^2)^{\frac{1}{4}} \sqrt{\frac{H}{\lambda}} \quad (16)$$

Thus, in the neighborhood of the origin exponent $n(x_{init})$ becomes

$$n(x_{init}) = n_1(x_{init}) + n_2(x_{init}) =$$

$$= 0.5 + 0.75\, p^2 + 0.871\, p(1 - p^2)^{\frac{1}{4}} \sqrt{H/\lambda} \quad (17)$$

As is seen from (17), in the neighborhood of the origin, the value of n increases according to a parabolic curve for an increase in the ratio H/λ. In Figure 2 the family of curves $n = n(H/\lambda)$ with parameter p is illustrated.

As is seen from the presented graph, for small H/λ's the value of $n\,(x_{init}) < n(x \gg 2H) = 2$ for any values of p. For large H/λ's and small p's the value of $n(x_{init})$ is also smaller than

$n(x \gg 2H)$. Consequently, in the case when the depth H is small in comparison with the length λ of the wave, the amplitude in the neighborhood of the origin decreases more slowly with distance than for greater distances of x; this is accurate for any ratio of velocity in the covering medium and in the refracting layer. For deep and large [waves] in comparison with long waves a similar regularity occurs for small values of p. For large H/λ and large p's the amplitude in the neighborhood of the origin decreases more steadily with distance than for large distances $x \gg 2H$.

Similarly from (2) there can be obtained a relationship n(x) for any values of $x > 2H \frac{p}{\sqrt{1 - p^2}}$. However, an investigation of this relationship presents numerous difficulties due to the fact that rays, in view of which the function $\frac{|F(\eta)|}{\eta^{3/2}}$ is presented, slowly come together. Now it is possible to estimate the behavior of the function A(x), given by formula (2), only for minimum and maximum potential values of x -- for the neighborhood of the origin and for x considerably larger than twice the depth of the separation boundary. A comparison of the values of n, obtained in both indicated cases, shows that for values of x not satisfying the inequality $x \gg 2H$, the value of n varies with a change of distance x. This result is necessary to allow for the development of a method to determine n according to experimental data.

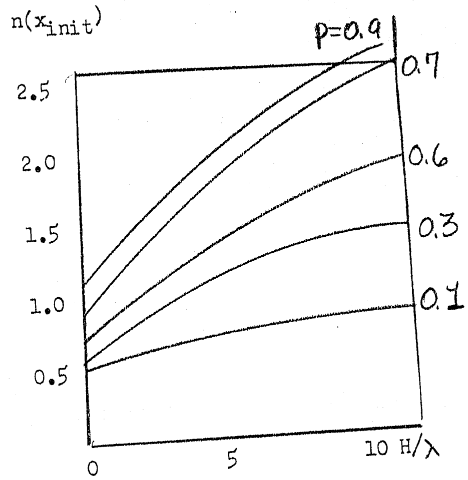

Figure 2. Curves of the ratio of $n(x_{init})$ to H/λ. Parameter of the family $p = \frac{V_1}{V_2}$ is the proportion of the velocity in the medium, covering refracting boundary and in the refracting layer.

3. The Approximate Formulas for Amplitude Curves of Refracted Waves

The amplitude curve in the case of a horizontal separation boundary.

Based on the theoretical works which were considered briefly in the preceding paragraph, ~~the ratio~~ of refracted wave amplitudes to distance in the case of a horizontal separation boundary of two non-absorbing media can be approximately presented in the form of the function

$$A(x) = \frac{C}{x^n} \tag{18}$$

where $x > x_{init}$, and n is the index of divergence, which according to the research results of different authors is of constant value, at least for distances of x which considerably surpass twice the depth of the separation boundary. Let us note that in some cases the layers of the medium of the divergence function, possibly, differ from exponential functions even when sufficiently removed from the origin.

Let us consider an approximation for rays in the cases of divergence functions of an exponential function of form (18); as will subsequently be seen, it confirms the results of the determination of n according to experimental data.

In the case of an absorbing medium with constant coefficients of absorption the variation of amplitude waves with distance, as was shown in the introduction, can be written in the following form:

$$A(x) = \frac{C}{x^n} e^{-\alpha_2 r_2 - \alpha_1 r_1} \qquad (19)$$

where C is a co[illegible] r_2 and r_1 are respectively the length of the path of the slid[illegible] refracted waves, α_2 and α_1 are the coefficients of abso[illegible] a refracting layer and in a covering medium, respectively [illegible]ase of a horizontal separation boundary r_1 = const; whe[illegible]or $e^{-\alpha_1 r_1}$, which determines the absorption in the pri[illegible] is a constant and it is possible for it to include the [illegible] The length of the path in the refracting medium $r_2 = x -$ [illegible]re H is the depth of the separation boundary, and i [illegible] is the critical angle. Thus, it is possible to wri[illegible]llowing form:

$$e^{-\alpha_2 (x - 2H \tan i)} \qquad (19')$$

In the semilogarithmic coordinate system (x, ln A) formula (19') becomes:

$$\ln A = -\alpha_2 x - n \ln x + C_1, \tag{20}$$

where C_1 is a constant depending on the value of C, H, i, and α_2. In the future we shall call the graphic relation $\ln A = \Psi(x)$ amplitude curves.

The Amplitude curve in the case of an inclined separation boundary. Using equation (20), let us determine the principle of variation of amplitude along the Ox line in the case of an inclined separation boundary. In Figure 3 the inclined separation boundary Q is shown and the trajectory of refracted waves for this case. The straight line Ox_1 is parallel to the boundary Q. In conformance with (20) the variation of amplitude along the line Ox_1 has the form

$$\ln A_1 = -\alpha_2 x_1 - n \ln x_1 + C_1 \tag{21}$$

where $x_1 > x_{init}$ is the distance reading from the point O along the line Ox_1.

Between the coordinates x_1 and x there exists the following relationship:

$$x_1 = x \frac{(\cos(i + \varphi))}{\cos i} \tag{22}$$

For the propagation of refracted waves between the straight lines Ox_1 and Ox a variation in the wave amplitudes at the expense of absorption and divergence in the primary medium occurs. It is possible to show that the divergence of the front of refracted waves in the area included between the straight lines Ox and Ox_1 can be disregarded since the effect of this factor on the decrease of wave amplitude is considerably smaller than the effect of absorption in the

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Substituting in (21) the coordinates x_1 for x, making allowances for power absorption in the primary medium on the path CE (Figure 3) and including the additive constant in the constant C_1, we obtain the following equation for the ratio of amplitude to distance x:

$$\ln A = - n \ln x - \alpha_2 x \frac{\cos(i + \varphi)}{\cos i} - \alpha_1 x \frac{\sin}{\cos i} + C_1 \quad (23)$$

where α_1 is the coefficient of absorption in the primary medium.

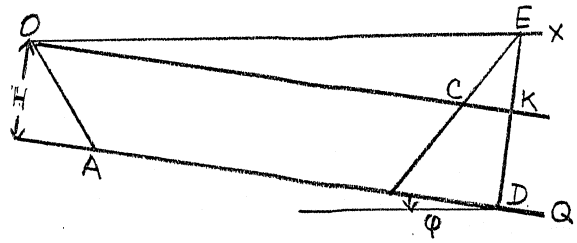

Figure 3. Deriving the equation of an amplitude curve for an inclined separation boundary

From (23) it is evident that for $\varphi < 0$, i.e. in the case where the profile is orientated in accordance with the slope of the separation boundary, the function $\ln A = \Psi(x)$ can have a minimum. The abscissa of the minimum point of the amplitude curve is expressed by formula as:

$$x_M = \frac{n \cos i}{\alpha_1 \sin \varphi - \alpha_2 \cos(i - \varphi)} \quad (24)$$

From (24) it is seen that the minimum point of the amplitude curve depends on the angle of slope φ of the separation boundary, on the size of the critical angle i, on the size of the absorption coefficients α_1 and α_2 in both media and on the index n of the divergence function.

The observed amplitude curve has a minimum in the case when the minimum point derived from formula (24) is located in an area where there are refracted waves. Moreover, this is necessary for the realization of the following conditions: the abscissa x_M of the minimum point must be greater than the abscissa $x_{init} = \frac{2H \sin i}{\cos(i - \varphi)}$ of the origin of the hodograph of a refracted wave and smaller than the abscissa x_{lim} of an end point, corresponding to the intersection of the separation boundary Q with the line Ox (Figure 4), i.e. the following inequality must be maintained:

$$\frac{2H \sin i}{\cos(i - \varphi)} < \frac{n \cos i}{\alpha_1 \sin \varphi - \alpha_2 \cos(i - \varphi)} < \frac{H}{\sin \varphi} \qquad (25)$$

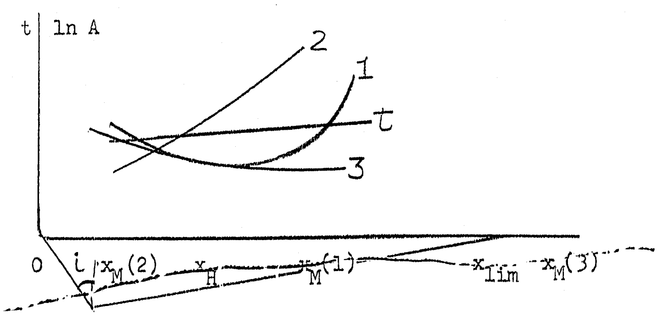

Figure 4. Hodograph and amplitude curves for the case of a profile situated in accordance with the eleveation of the refracting boundary. Curve 1 -- $x_{init} < x_M < x_{lim}$, Curve 2 -- $x_M < x_{init}$, Curve 3 -- $x > x_{lim}$

If condition (25) is observed, then the curve will have the form of curve 1 shown in Figure 4. If $x_M < x_{init}$, which for the most part is possible in the case of a deep separation boundary when x_{init} is of a comparatively large size then the amplitude will be a monotonic increasing function of the distance x (Figure 4, Curve 2). Finally, if $x_M > x_{init}$, which is possible for small angles of slope and small absorption coefficients α_1 and α_2, then the amplitude will be a monotonic decreasing function of the distance x (Curve 3).

Knowing the values of i, α_1, α_2, and n, it is possible to determine the range of the values of H and φ for which each one of the three cases analyzed will occur. In Figure 5 a nomographic chart is presented which permits determination of the three ranges specified above for the particular case when n = 1,5, $i = 30^\circ$, $\alpha_1 = 0.05\ m^{-1}$, $\alpha_2 = 0.005\ m^{-1}$.

In the nomograph are shown three families of straight lines with parameter φ; I is $x_{init} = \frac{2H \sin i}{\cos(i - \varphi)}$; II is $x_{lim} = \frac{H}{\sin \varphi}$ and III is $x_M = \frac{n \cos i}{\alpha_1 \sin \varphi - \alpha_2 \cos(i - \varphi)}$. Each straight line of family I has the geometric locus of the origin for a fixed angle φ. The range of values x and H for which there can be recorded refracted waves, characteristic of the separation boundary with fixed angles of inclination φ, [redacted] are located above the straight line with fixed value of the parameter φ.

The straight line family II is the geometric locus of the intersection points of the beginning of the separation boundary with the horizontal terrestrial surface. The range of values x and H for which there may exist refracted waves characteristic of the separation boundary with different angles of inclination φ, is located between the lines of family I and II with similar values of the parameter φ.

The straight line family III is the geometric locus of the minimum point of the amplitude curves $\ln A = f(x; \varphi)$, corresponding to the separation boundary with fixed values of the angle of inclination φ. Each family of straight lines III with a fixed parameter φ divides the area of existing refracted waves into two parts. The waves included between I and II have similar values of the parameter φ. In the range of values x and H, which are located above the straight line family III, the amplitude of a refracted wave grows with an increase in x, but in the range of values x and H which lie below the straight line family III, the amplitude decreases with an increase in x. For values of H smaller than the abscissa H_a intersecting straight line families II and III with equal values of parameter φ, the amplitude is monotonic increasing with the distance x for all possible values of x. For values of H greater than the abscissa H_b intersecting the straight line families I and III with equal values of φ, the amplitude for all distances increases monotonically with the value of x. For values of H which satisfy the inequality $H_a < H < H_b$, the amplitude curve has a minimum.

An analysis of Figure 5 shows that for assumed values of α_1, α_2, n and i and for small values of the angle of inclination φ the range of values of x and H for which the amplitude is monotonic increasing with distance is comparatively large, and only for greater distances of x does the amplitude reach a minimum and after this it increases with distance. There can only be a monotonic increase of wave amplitude in all lengths of the line observed for sufficiently great depths H. As, for example, for $\varphi = 8^{\circ}$, the amplitude is monotonic decreasing with distance for H 80 and for all possible values of x. For 80 meters $< H <$ 560 meters the amplitude decreases for all values of x $<$ 560 meters; for x = 560 meters the amplitude curve has a minimum and for further values of x the amplitude increases with distance. For H $>$ 560 meters the amplitude increases with distance for all possible values of x.

For large angles of inclination φ the range of x and H, for which the amplitude is monotonic decreasing with distance, is extremely small; as for $\varphi = 10^\circ$, the amplitude is monotonic decreasing only for H $\leqslant$ 50 meters; for 50 meters $< H <$ 240 meters the amplitude curve has a minimum relatively near the distance B = 250 meters from the epicentre and for H $>$ 240 meters the amplitude monotonically increases with distance for all values of x.

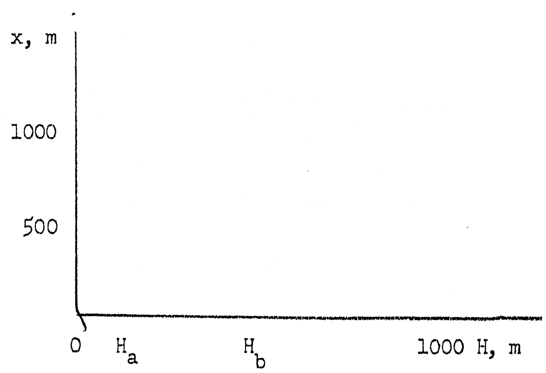

Figure 5. Nomographic chart for the determination of the character of an amplitude curve inthe case of a profile which lies on the slanting ~~rise of~~ the refracting boundary: the dotted lines are family I, $x_{init} = \frac{2H \sin i}{\cos(i-\varphi)}$; the unbroken line is family II, $x_{lim} = H \sec \varphi$; the dotted line with points is family III, $x_M = \frac{n \cos i}{\alpha_1 \sin \varphi - \alpha_2 \cos(i-\varphi)}$. For $\varphi = 8^\circ$ the vertical shading shows the range of values of x and H, where the amplitude increases with distance x, and the slanting shading is the range where the amplitude increases with distance.

The Amplitude Curve in the Case of a Curvilinear Separation Boundary. In the case of a curvilinear separation boundary for relatively small curvature the equation of an amplitude curve can approximately be represented in a form similar to (23). The basic differences are the following: (1) for constant velocity of the characteristic medium the index n in the case of a curvilinear separation boundary may essentially be distinguished from the index of a flat separation boundary; (2) the length of the wave path in the refracting medium must be read along a curvilinear line, and not on a rectilinear as on a refracting boundary; (3) the length of the wave path in the primary medium varies in accordance with a more complex principle than in the case of a rectilinear boundary.

The equation of the amplitude curve in question can be presented in the following form:

$$\ln A = - n \ln r - \alpha_2 (r - s_2) - \alpha_1 s_1 + C, \qquad (26)$$

where r is the distance read along the curvilinear refracting boundary from the basic normal which was dropped from the epicentre onto the separation boundary to the discharge point of seismic radiation from the second medium, s_2 is the length of the curvilinear boundary segment from A normal to point B of seismic radiation discharge in the second layer.

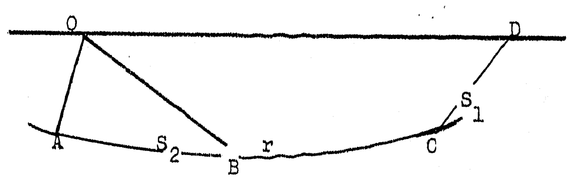

Figure 6. The development of the relation of the amplitude curve for a curvilinear separation boundary

4. The Processes for Determining the Index of Divergence from Experimental Data

Let the index of divergence n and the absorption coefficients α_1 and α_2 be constants. From these assumptions the determination of values for n can be derived by two methods: (1) according to a system of two overtaking [redacted] amplitude curves and (2) according to a system of two counter [redacted] amplitude curves. Both methods are based on the hypothesis that the waves, recorded at different epicentres, are characterized by the same predominant frequency of oscillation and while the absorption coefficients α_2 in the equations of amplitude are applicable to different epicentres, it is possible to regard them as equal. This is based on the separate investigation by this method of each of them.

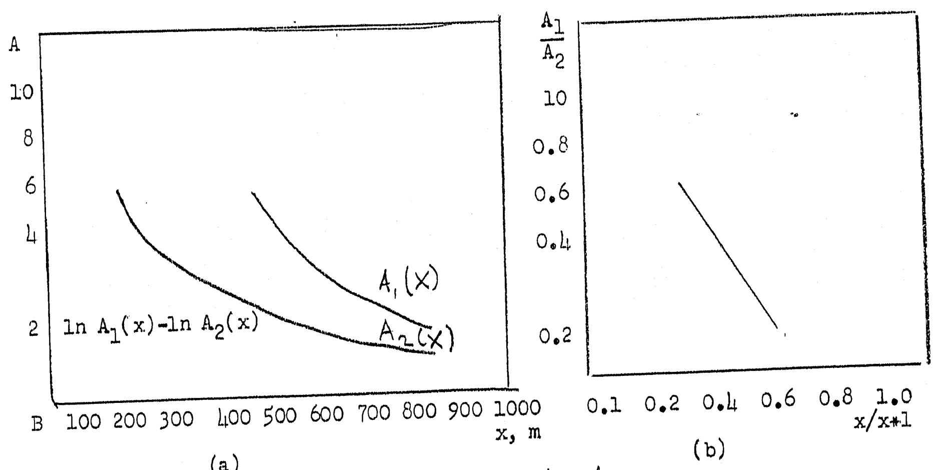

Figure 7. The determination of n by two overlapping [redacted] amplitude curves: (a) the overlapping [redacted] amplitude curves $A_1 = A_1(x)$ and $A_2 = A_2(x)$; (b) the curve ln A_1/A_2 = $\Psi(x/(x + \ell))$ in the u, w coordinate system.

The Determination of n According to a System of Two [redacted] Over-lapping [redacted] Amplitude Curves. Let us consider at first the case of a flat inclined refracting boundary. Let the equation of the amplitude curve obtained at the explosions in point 1 (Figure 7) have the form:

$$\ln A_1 = -\alpha_2 x \frac{\cos(i+\varphi)}{\cos i} - n \ln x - \alpha_1 x \frac{\sin}{\cos i} + C_1. \qquad (27)$$

The equation of the amplitude curve obtained at explosions in point 2 which is found by the distance ℓ from point 1, has the form:

$$\ln A_2 = -\alpha_1 (x + \ell) \frac{\cos(i+\varphi)}{\cos i} - n \ln (x + \ell) - \alpha_1 (x + \ell) \frac{\sin\varphi}{\cos} + C_2 \qquad (28)$$

From formulas (27) and (28) it is possible to obtain the following equation for the difference $\ln A_1 - \ln A_2 = \ln A_1/A_2$:

$$\ln \frac{A_1}{A_2} = -n \ln \frac{x}{x+\ell} + \alpha_2 \ell \frac{\cos(i+\varphi)}{\cos i} + C. \qquad (29)$$

Let us introduce a new coordinate system:

$$u = \ln \frac{x}{x+} \qquad w = \ln \frac{A_1}{A_2} \qquad (30)$$

The difference curve (29) in this coordinate system is transformed into a straight line

$$w = -nu + C_3, \qquad (31)$$

the angular coefficient of which, taken with opposite signs, is equal to the index of divergence n. This construction can be useful in effecting the following form. According to experimental converging amplitude curve fitting, constructed on a semilogarithmic grid (x, $\ln A$), it is possible for each one of the given values of x to represent graphically the difference $\ln A_1 - \ln A_2 = \ln A_1/A_2$ and to plot the changes of value of $\ln A_1/A_2$ on the two-part semilogarithmic grid (Figure 7 b), in which are double scales along the axis of the abscissa [x-axis] (logarithmic for $\frac{x}{x+\ell}$ and equal for $u = \ln \frac{x}{x+\ell}$) and the axis of the ordinate (logarithmic for A_1/A_2 and equal for $w = \ln \frac{A_1}{A_2}$). Approximating the plotted points of the straight line and determining for this straight line the values $\Delta u = u_k - u_i$ and $\Delta w = w_i - w_k$, it is possible to compute $n = \frac{\Delta w}{\Delta u}$. In the case of a curvilinear separation boundary, which is characterized by comparatively little curvature, formula (31) is still correct, only in this case the abscissa u has the form

$$u = \ln \frac{r}{r+p}\,; \qquad (30')$$

Here p and r are distances shown in Figure 8. For determinations of n in this case it is necessary first of all to plot according to the hodograph of the refracting boundary and make the distance readings along the curvilinear boundary.

The Determination of n by Counter [redacted] Amplitude Curves. For two Counter [redacted] amplitude curves, we have the equations:

$$\ln A_1 = -\alpha_2 x \frac{\cos(i+\varphi)}{\cos i} - n \ln x - \alpha_1 x \frac{\sin\varphi}{\cos i} + C_1 \qquad (32)$$

and

$$\ln A_2 = -\alpha_2 (a-x) \frac{\cos(i-\varphi)}{\cos i} - n \ln(a-x) + \alpha_1 (a-x) \frac{\sin\varphi}{\cos i} + C_2 \qquad (33)$$

where a is the distance between the two epicentres (Figure 9). Adding (32) and (33), we obtain the equation of added curves:

$$\ln A_1 + \ln A_2 = \ln (A_1 A_2) = \frac{2 \sin \varphi}{\cos i} \; x(\alpha_2 \sin i - \alpha_1) - n \ln x(a-x) + C. \tag{34}$$

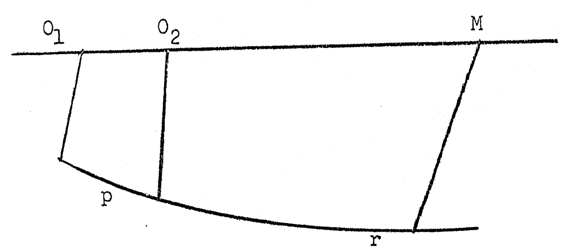

Figure 8. Determining n by overlapping [redacted] amplitude curves when there is a curvilinear separation boundary

In the case of a horizontal separation boundary $\varphi = 0$ and

$$\ln (A_1 A_2) = - n \ln x(a - x) + C. \tag{35}$$

Let us present the coordinate system:

$$y = \ln x(a - x), \qquad z = \ln (A_1 A_2). \tag{36}$$

In this coordinate system, curve (35) transforms into a straight line

$$z = -ny + C, \tag{37}$$

the angular coefficient of which, taken with opposite signs, equals n (Figure 9b).

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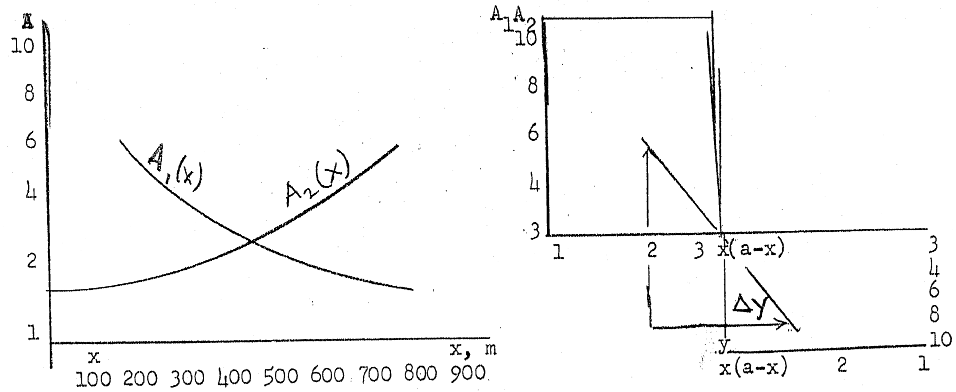

Figure 9. Determining n according to Counter [redacted] amplitude curves: (a) Counter [redacted] amplitude curve $A_1 = A_1(x)$ and $A_2 = A_2(x)$; (b) curve $\ln (A_1A_2) = \Psi [x(a - x)]$ in the (y, z) coordinate system

For a determination of n according to the system of two Counter [redacted] amplitude curves, it is necessary to have in mind that (y, z) coordinates of the points of the straight line (37) conform to the values of x and $x_1 = a - x$, and consequently they correspond to segments of the straight lines corresponding to values of x in the interval $0 < x < a/2$ and $a/2 < x < a$. In connection with this, for convenience of representation, it is expedient to plot the points which comprise the distance interval $0 < x \leq a/2$, but then, turning the grid around the z axis and the y_1 axis $= \ln (A_1A_2)\Big|_{x = a/2}$, to plot the points which comprise the interval $a/2 < x < a$. Drawing the straight line which approximates the plotted points, and determining from this line the values $\Delta y = y_k - y_i$ and $\Delta z = z_i - z_k$ (Figure 9b), it is possible to compute the value of n, $n = \Delta z/\Delta y$.

Comparison of Methods of Determining n by Overlapping [redacted] and [redacted] Counter [redacted] Amplitude Curves. The method of determining n by overlapping [redacted] curves is essentially opposed to the method of determination by counter curves. It is clear that for the construction of amplitude differences the deviations of separate points of the curves (these deviations being dependent upon the variations in seismographic equipment along the profile line or the differences in passage sensitivity) are excluded. It is necessary to emphasize that in order to employ the method of overlapping [redacted] waves there is no need for a precise selection of equal passage sensitivity and like specifications for seismographic equipment along the profile; for a determination of n it is necessary only that the passage sensitivities and seismograph equipment remain unchanged when obtaining recordings from both epicentres.

In using the method of counter curves the deviation of points of the amplitude curves are dependent upon the differences in passage sensitivities and condition of equipment which together [redacted] distort the results of determinations of the values of n. Let us note also that the method of diverging curves gives inaccurate results in the case of slanted planes and curvilinear separation boundaries, as the method of overlapping curves principally gives accurate results for any types of refracting boundaries if the granted assumptions concerning the constancy of n, α_1 and α_2 are observed. Thus, the method of overlapping curves is more accurate and easier for practical application. It is expedient to use the method of counter curves only in those cases when overlapping curves are absent. In the future we shall investigate only the method of overlapping curves.

5. The Applicability of the Method of Overlapping [redacted] Curves when n and α_2 are Variables

In the preceding paragraph the method of determination by two converging amplitude curves based on the hypothesis that n and α_2 are constants was investigated. In this paragraph we shall study the problem of the applicability of this method when either n or α_2 is a variable.

The Index n is a Function of Distance from an Epicentre. As was shown in Paragraphs 1 and 2, from the investigation of Brekhovskikh's formula [11] it follows that the index n does not depend on the distance x only for values of x which are considerably greater than twice the depth of the separation boundary. For smaller values of x, the index n depends on distance, i.e., $n = n(x)$. Let us investigate the problem of possible errors in determining n by the method of overlapping [redacted] curves when n depends on the distance between the observation point and the oscillation source. In this case formula (29) becomes:

$$\ln \frac{A_1}{A_2} = -n(x)\ln x + n(x+\ell)\ln(x+\ell) + C \qquad (38)$$

In the (u, w) coordinate system, curve (38) is transformed into a straight line when n is a constant, but otherwise into a curve. The effective value of the index of divergence n_{eff} numerically will be equal to the angular coefficient of the tangent to this curve, given with opposite signs:

$$n_{eff} = -\frac{dw}{du} = n(x) - \underline{x}[n(x+\ell) - n(x)]. \qquad (39)$$

Introducing the specifications

$$n_{eff}(x) - n(x) = \Delta n_{eff}(x),$$
$$n(x + \ell) - n(x) = \Delta n(x),$$

it is possible to set up (39) in the following form:

$$n_{eff}(x) = -\frac{x}{\ell} \Delta n(x). \qquad (39')$$

Thus, the variation of the value of $n_{eff}(x)$ from $n(x)$ is equal to the difference of the value $n(x + \ell)$ and $n(x)$, given with opposite sign and multiplied by the coefficient x/ℓ.

From formulas (39) and (39') it is seen that when $\Delta n(x) < 0$, i.e. when $n(x) > n(x + \ell)$, the inequality is correct

$$n_{eff}(x) > n(x) > n(x + \ell); \qquad (40)$$

when $\Delta n(x) > 0$, i.e. when $n(x) < n(x + \ell)$, this inequality holds

$$n_{eff}(x) < n(x) < n(x + \ell). \qquad (41)$$

Consequently, in both cases the value which is determined for n_{eff} is not intermediate between the values of $n(x)$ and $n(x + \ell)$. Depending on the relation of $n(x)$ and $n(x + \ell)$ the value of $n_{eff}(x)$, which is determined by the slope of the tangent to curve (38), will be either larger than the larger of the two values $n(x)$ and $(n)(x + \quad)$, or smaller than the smaller of these two values. From the study introduced in Paragraph 2, of the Brekhovskikh formula, for x larger than 2H and for the neighborhood of the origin it follows that relationship (40) must occur for larger values of H/λ and large values of the velocity rate p, but relationship (41) is for large values of H/λ for small p or for small H/λ for any value of p.

From equation (39) it is seen that the value of n_{eff} varies with distance x, during which its nature of change depends on the form of the function $\Delta n(x) = n(x + \ell) - n(x)$, which up to now has not been investigated. For distances $x \gg 2H$, as follows from the theoretical work of Brekhovskikh and other authors the function $\Delta n(x)$ converges to zero, and the value of n_{eff} converges to its real value n. The greatest distortion in determining the index n, which is dependent upon its variation with distance, must occur for small distances of x, i.e. in those cases when amplitude waves obtained near the origin are used for determining n. Consequently, for determining n it is expedient to select a section of the amplitude curves which is sufficiently far from the origin.

The Variable α_2. For a study of the α_2 variables it is necessary to set forth two cases: (1) the absorption coefficient depends on the distance between the point of the refracting boundary and the epicentre and (2) the coefficient of absorption depends on the nature of the refracting layer in different points of the separation boundary and does not depend on the distance between these points and the epicentre. For brevity, in the first case, we shall say that α_2 is a function of distance from the epicentre, and in the second case, that α_2 is a function of the coordinates of a point of the refracting boundary.

1. The value of α_2 may be a function of distance from the epicentre in this case, if wave penetration in the second medium occurs or if the majority of waves vary with distance. It is known that in the neighborhood of the refracting layer, in particular for stratification with little depth, rocks often are changed by the processes of weathering and oxidation. This should lead to some in-

creased values of the absorption coefficient α_2 in the neighborhood of this layer and to the presence of a negative vertical gradient of the value α_2. One may suppose that at some depth H_i in the refracting layer, if it is formed of similar rocks, a constant value of α_2 is determined. For small distances x the wave propagates in a neighborhood of the refracting layer where the value of α_2 is somewhat increased, but for great distances the wave penetrates the depth of the refracting layer and beginning with some distance x, attains the depth H_i at which an approximately constant value of α_2 is established. Consequently, in the presence of penetration, the value of α_2 must be a decreasing function of distance.

Let us note that as regards the presence of penetration it is not always possible to pass judgment on the basis of studying the form of the refracted wave hodograph, since for a small vertical gradient of velocity, penetration may have practically no effect on the parallelism of the converging hodograph. Even now for a small gradient α_2 the penetration may apparently exert considerable influence on the path of the amplitude curve and, in conjunction with this, on the accuracy of the determination of values of n.

The decrease in the coefficient of absorption α_2 for increased distances from the epicentre may also be dependent upon the variation with distance of the predominant frequencies of recorded waves. In several cases, particularly in the presence of thin refracting layers, lying at small depths, predominant frequencies of refracted waves vary sharply with distance at a small withdrawal from the epicentre and, only beginning with such a distance, an approximate constancy of the predominant frequency is set up. In these cases, in the presence of two overlapping [redacted] epicentres the predominant wave frequency, recorded at the nearer epicentre, is greater than for the more removed epicentre. In relation to this, the values of α_2 will also be differentiated since the coefficient of absorption α_2 increases with frequency. Consequently, for a variation of the predominant frequencies of waves with distance the coefficient of absorption α_2, just as in the case of penetration, will be decreasing functions of distance.

2. The value of α_2 is a function of the coordinates of a point as long as the physical properties of refracting layer II change along its boundary. This may occur during a change of the lithologic structure of rocks which are a part of medium II. In particular, such a change is possible in this case if, in medium I, there are vertical separation boundaries of the layers with different deflection distances and different coefficients of absorption. In some cases there may be a change of the coefficient of absorption in the same rocks if they have areas with different porosity and fracturing.

The Coefficient of Absorption α_2 as a Function of Distance from an Epicentre. Let us present the problem of the accuracy of determining n according to the method of overlapping curves, if α_2 is a function of distance from an epicentre. On the basis of the physical conditions given previously we shall assume that is a decreasing function of distance, which is read along the refracting boundary and, consequently, for $r_2' < r_2''$, $\alpha_2(r_2') > \alpha_2(r_2'')$.

The equation of amplitude curves in this case will be

$$\ln A_1 = - n \ln x - \alpha_1 x \frac{\sin}{\cos i} - \int_{H_1 \tan i}^{r_2'} \alpha_2(r_2) dr_2 + C_1, \quad (42)$$

$$\ln A_2 = -n \ln (x + \ell) - \alpha_1 (x + \ell) \frac{\sin \varphi}{\cos i} - \int_{H_2 \tan i}^{r_2''} \alpha_2 (r_2) dr_2 + C_2 \qquad (43)$$

where H_1 and H_2 are depths in respect to the normal from the epicentres 1 and 2, respectively, $r_2' = x \frac{\cos(i+\varphi)}{\cos i} - H_1 \tan i$, $r_2'' = (x + \ell) \frac{\cos(i+\varphi)}{\cos i} - H_2 \tan i$. The difference $\ln A_1 - \ln A_2 = \ln A_1/A_2$ can be represented in the following form:

$$\ln \frac{A_1}{A_2} = -n \ln \frac{x}{x + \ell} - \int_{H_1 \tan i}^{r_2'} \alpha_2 (r_2) dr_2 + \int_{H_2 \tan i}^{r_2''} \alpha_2 (r_2) dr_2 + C \qquad (44)$$

Let us introduce the notation

$$\int_{H_1 \tan i}^{r_2'} \alpha_2 (r_2) dr_2 - \int_{H_2 \tan i}^{r_2''} \alpha_2 (r_2) dr_2 = \varphi (x) \qquad (45)$$

Thence (44) appears as

$$\ln \frac{A_1}{A_2} = -n \ln \frac{x}{x + \ell} - \varphi (x) + C, \qquad (46)$$

or in the (u, w) coordinate system

$$w = -nu - \varphi (u) + C. \qquad (47)$$

The effective value of n_{eff} is numerically equal to the angular coefficient of the tangent to curve (46), with opposite sign. The formula for n_{eff} will be

$$n_{eff} = -\frac{dw}{du} = n + \frac{d\varphi}{dx} \frac{dx}{du} \qquad (48)$$

Inasmuch as $u = \ln \frac{x}{x + \ell}$, then the derivative

$$\frac{dx}{du} = x\left(\frac{x}{\ell} + 1\right) > 0 \qquad (49)$$

It is possible to express the derivative $d\varphi/dx$ in the following manner

$$\frac{d\varphi}{dx} = \frac{\partial\varphi}{\partial r'_2}\frac{dr'_2}{dx} + \frac{\partial\varphi}{\partial r''_2}\frac{dr''_2}{dx} = \frac{\cos(i + \varphi)}{\cos i}\,[\alpha_2(r'_2) - \alpha_2(r''_2)]. \qquad (50)$$

According to the condition that $\alpha_2(r'_2) > \alpha_2(r''_2)$, then $\frac{d\varphi}{dx} > 0$. The product $\frac{d\varphi}{dx} \cdot \frac{dx}{du} > 0$; consequently, for the given hypothesis regarding the dependence of α_2 on distance, the value of n_{eff} which is restricted according to formula (48), is greater than the true value of n.

Introducing the specification

$$\alpha_2(r'_2) - \alpha_2(r''_2) = \delta\alpha_2 \qquad (51)$$

and taking into account equation (49) for dx/du, let us write the equation for the error in determining n in the following form:

$$\delta n = \delta\alpha_2\left[x\left(\frac{x}{\ell} + 1\right)\frac{\cos(i + \varphi)}{\cancel{\cos i}}\right] \qquad (52)$$

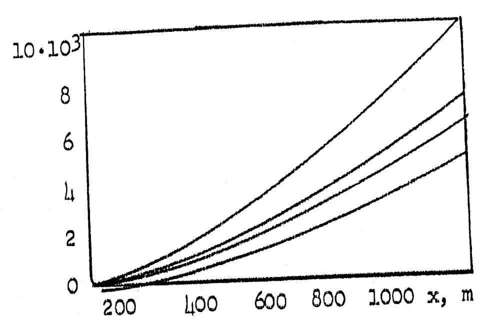

Figure 10. Curves of the ratio of $\delta n/\delta\alpha_2$ to x; the parameter of the family is the distance ℓ between epicentres.

From formula (52) it is seen that δn is greater, the greater the value of x and the smaller the value of $\delta\alpha_2$. It is necessary to keep in mind that the value, in turn, is a function of x and ℓ. The difference $\delta\alpha_2$ may attain large values for small x/ℓ; for large x/ℓ one may assume that $\delta\alpha_2$ will approach zero. In Figure 10 is shown the family of curves $\delta n/\delta\alpha_2 = \delta\, n(x, \ell)$ calculated for different values of the parameter ℓ of the horizontal separation boundary. As is seen from the graph, even comparatively small variations in the value of α_2 cause considerable errors in the determination of n. As, for example, for x = 400 meters, ℓ = 1000 meters and $\delta n/\delta\alpha_2$ = = 560; if assuming that $\delta\alpha_2$ is small, being equal to 0.001 m^{-1}, the error δn = 0.56. For x = 600 meters, ℓ = 1000 meters and $\delta\alpha_2$ = 0.001 m^{-1}, the error δn = 0.96. Insofar as the occurence of penetration may take place essentially at distances close to the epicentre, then for greater accuracy in determining n, it is expedient to make use of observation areas not too near the explosion point. In these cases one may assume that waves, excitable in both epicentres are propagated along one and the same boundary of separation, and therefore $\delta\alpha_2$ = 0 and, consequently, δ n = 0.

As was shown earlier, for a decrease of frequency with distance, as in the presence of penetration, the coefficient of absorption also decreases with distance. In this case errors in determining n also are contained in formula (52). As was shown, the results of treating field experimental data, obtained at the Geophysics Institute in 1946, for solid limestone with a velocity V = 400 meters per second, for frequency variations from 60 to 100 megacycles, the coefficient of absorption varies by 0.003 - 0.006 m^{-1}. For less solid rocks the frequency variation may specify considerably greater values of $\delta\alpha_2$ which essentially represent, in accuracy, determinations of n.

From what has been stated, it is seen that wave penetration in a medium which is located below the refracting boundary, and the decrease of the prevailing wave frequency with distance lead to an exaggeration of the value of n determined by two overlapping amplitude curves. Therefore, for determining n it is necessary, in the first place, to use only recordings with identical predominant oscillation frequencies and, in the second place, to select regions of amplitude curves which are obtained at sufficiently great distances from the epicentre. In this case one may expect that penetration would not play an important part.

The Appearance of Penetration and the Variation in the Value of n with Distance. From the preceding investigation it follows that, just as n varies with distance from an epicentre, i.e. for $n = n(x)$, so α_2 varies with distance from an epicentre according to the specified penetration, i.e. for $\alpha_2 = \alpha_2(x)$, the method of overlapping curves gives distorted values of n_{eff}. The question arises whether according to observed data it is possible to set up the relation $n = n(x)$ and $\alpha_2 = \alpha_2(x)$ and whether it is possible to limit these two phenomena.

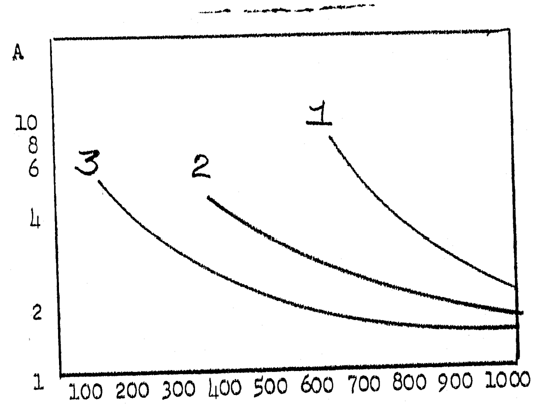

Figure 11. Determining n by three overlapping amplitude curves

In both cases illustrated the value of n_{eff} must depend on at what distances from the epicentre are obtained the recordings according to which the regions of amplitude curves which are to be used are plotted. Consequently, if the value of n_{eff}, determined by different pairs of amplitude curves, obtained from one and the same region under conditions of constant predominant oscillation frequencies, is different, then it indicates that there occurs, at any rate, one of the two phenomena under investigation. In principle, the simplest systems observed, permitting the establishment of the ratio of n_{eff} to distance x, is the system of three overlapping amplitude curves (Figure 11). In some cases a comparison of the values obtained from different pairs of curves may permit solution of the problem as to whether the difference in the values of n_{eff} is dependent upon penetration, i.e. the relation $\alpha_2 = \alpha_2(x)$, or $n = n(x)$.

If n is a monotonic increasing function of x, then for determination according to curves 1 and 2 (Figure 11) there will be obtained a smaller value of n_{eff} than for determination by 2 and 3, i.e. n_{eff} will increase with distance x. At the beginning of penetration, the value of n_{eff}, as pointed out earlier, decreases with an increased x. Consequently, on determining n by different pairs of amplitude curves it is possible to distinguish the case when n is a monotonic increasing function of distance from the case of penetration.

If n is a monotonic decreasing function of distance, then the value of n determined by curves 1 and 2 (Figure 11) will be larger than n_{eff}, as determined by curves 2 and 3, i.e. a result will be obtained similar to the one at the beginning of penetration. Consequently, by the results of determining n_{eff} by some overlapping curves, it is impossible to distinguish the case of a monotonic increasing function of $n(x)$ from the case of penetration.

The Coefficient of Absorption α_2 is a Function of the Coordinates of a Point of the Refracting Boundary. Let us study the problem of the possibility of determining n by the method of overlapping curves when is a function of the coordinates of a point on the refracting boundary. For the purposes of this study let us disregard the complicated factors in the path of amplitude curves which may occur near the separation boundary layers with different velocities and coefficients of absorption, and we shall assume that in the given case the amplitude is a continuous function of distance. Then, if for each pair of values r_2' and r_2'', the equality $\alpha_2(r_2') = \alpha_2(r_2'')$, then in formula (50) $d\varphi/dx = 0$, and therefore, $n_{eff} = n$.

Thus, we use the method of determining n by overlapping amplitude curves not only when α_2 is a constant, but also when α_2 is a variable depending on the coordinates of a point of the refracting boundary.

6. The Relation of Accuracy in Determining the Coefficient of Absorption to the Accuracy of Determining the Power Exponent n

The Methods of Determining the Coefficient of Absorption α_2. In the work of Yu. I. Vasil'yev [14] methods are proposed for determining the coefficient of absorption for the case of a rectilinear separation boundary and for a separation boundary of an arbitrary form. We shall be arrested in the analysis of the accuracy in determining only for the case of the rectilinear separation boundary.

The Vasil'yev method is based on the transformation of observed amplitude curves $\ln A = f(x)$ into curves of the form $\ln Ax^n = \psi(x)$.

For this transformation it is assumed that the index n is given.

In the case of a horizontal separation boundary the amplitude curve, as shown in formula (20) is transformed into the straight line

$$\ln Ax^n = - \alpha_2 x + C, \qquad (53)$$

the angular coefficient of which, taken with opposite sign, equals α_2. In the case of an inclined separation boundary the angular coefficient of the tangent to the transformed curve, as seen from formula (23) depends on the coefficients of absorption α_1 and α_2, slope φ and on the critical angle i. To determine α_2 in this case, Vasil'yev recommends changing to the method of difference which results in two counter amplitude curves which graphically represent the difference of the values:

$$\ln A_2(a - x)^n - \ln A_1 x^n = Q(x) \qquad (54)$$

When α_2 = const the graph of the difference is a straight line, the angular coefficient of which is numerically equal to $2\alpha_2 \cos\varphi$. Knowing the slope φ, it is possible to determine the coefficient of absorption α_2. If the value of α_2 is different in different regions of the profile, then the graph of the difference will be represented as broken lines upon which the slope of the separate segments of the broken lines will be changed in ratio to the value of α_2.

The method of difference permits us to exclude divergence of separate points of the amplitude curves dependent upon dissimilar excitable channels and variations of equipment condition. In this respect the method of determining α_2 by counter curves is analogous to the method of determining n by overlapping curves (see Paragraph 4). This substantial advantage of determining α_2 by a single curve causes us to regard its use as expedient not only when the separation boundary is inclined but also when it is horizontal.

Errors in Determining α_2 by the Method of Difference as Caused by Errors in the Value of n. As a result of determining the deduced effective value, let $n_{eff} = n + \Delta n$, where n is the true value of the index of divergence and Δn is the error. In this case the graph of function (54) will be a curve of the following form:

$$y = \ln A_2 (a - x)^{n_{eff}} - \ln A_1 x^{n_{eff}} = 2\alpha_2 x \cos\varphi + \Delta n \ln \frac{a - x}{x} \quad (55)$$

For determining $\alpha_{2_{eff}}$ it is necessary to find the angular coefficient of the tangent to curve (55) which is expressed by the formula:

$$\frac{dy}{dx} = 2\alpha_{2_{eff}} \cos\varphi = 2\alpha_2 \cos\varphi - \Delta n \frac{1}{x\left(1 - \frac{x}{a}\right)} \quad (56)$$

The error in determining α_2 equals

$$\Delta\alpha_2 = \alpha_{2_{eff}} - \alpha_2 = -\frac{1}{2\cos\varphi} \Delta n \frac{1}{x\left(1 - \frac{x}{a}\right)} \quad (57)$$

As is seen from equation (57), the sign of the error in determining α_2 is opposite to the sign in determining n. The function $\Delta\alpha_2/\Delta n = f(x)$ has a minimum at $x = a/2$ and is a curve symmetric with respect to the line $x = a/2$. The value $\left(\frac{\Delta\alpha_2}{\Delta n}\right)_{min} = -\frac{2}{a\cos\varphi}$, i.e. it is the inverse of the proportional distance a ~~between both~~ epicentres.

In Figure 12 when $\varphi = 0$ a graph of the ratio of the value $\Delta\alpha_2/\Delta n$ to distance x for different values of a is presented. As is seen from the study of the graph the error in determining α_2 attains the greatest value and sharply changes with distance for small x's and for x near a, i.e. in those cases when regions of amplitude curves near each epicentre are used for determinations. In the area of the value of x near a/2, i.e. in the area of minimum curve $\Delta\alpha_2/\Delta n = f(x)$ at which the region of minimum curve is wider, the greater is a.

Consequently, when a is small for sufficiently large errors in determining n curve (55) cannot be approximated as a straight line in a sufficiently extended region. In the case of comparatively larger values of $a > 400$ meters, curve (55) to a sufficient extent with a comparatively great degree of accuracy can be approximated as a straight line with angular coefficient equal to $2\cos(\alpha_2 + \overline{\Delta\alpha_2})$, where $\overline{\Delta\alpha_2}$ is the mean value of error in the region of minimum curve $\Delta\alpha_2/\Delta n = f(x)$. From this it follows that for a study of small depths, when distance a between the counter epicentres is sufficiently small, for a reliable determination of α_2 especially great accuracy in determining the index of divergence n is necessary. For non-observance of this condition the value of error $\Delta\alpha_2$ may be of such size as the very variation of α_2, and in some cases even more. As, for example, from a study of Figure 12, it is seen that when a = 100 meters, $\left(\frac{\Delta\alpha_2}{\Delta n}\right)_{min} = -0.02\ m^{-1}$, $\Delta n = 0.5$, $(\Delta\alpha_2)_{min} = -0.01\ m^{-1}$, i.e. $\Delta\alpha_2$ attains the same magnitude as the value of α_2 in some sedimentary and metamorphic rocks. For a = 500 meters and $\Delta n = 0.5$ the error is considerably decreased and the value of $(\Delta\alpha_2)_{min} = 0.003/m^{-1}$. Thus, the accuracy of determining the value of α_2 depends essentially on the accuracy of determining the index n and on the distance a between both epicentres.

The Error in Determining α_2 by a Single Amplitude Curve. Earlier it was pointed out that the determination of by a single amplitude curve principally gives accurate results only in the case of a horizontal separation boundary whereupon even for such a simple form of this boundary it is more expedient to substitute the method of difference of two counter curves for exclusion of the effect of equipment conditions and the inaccuracy of the regulation of excitability of different channels. However, for the treatment of experimental data sometimes the counter amplitude curves are absent, and the value of α_2 has to be determined from one curve. In relation to this it is expedient to study the problem of errors in determining α_2, which are dependent upon the errors in the determination of n, by a single curve. Let us study the case of the horizontal separation boundary.

If the error in the determination of n equals Δ n, then formula (55) takes the form:

$$y = \ln Ax^n = -\alpha_2 x + \Delta n \ln x \qquad (58)$$

The angular coefficient of the tangent to curve (58), which is numerically equal to the value of $\alpha_{2_{eff}}$, with opposite sign, will be

$$-\alpha'_{2eff} = -\alpha_2 + \frac{\Delta n}{x} \qquad (59)$$

$$\Delta\alpha'_2 = \alpha_{2_{eff}} - \alpha_2 = -\frac{\Delta n}{x} \qquad (60)$$

Equation (60) along with (57) for the case $\varphi = 0$ shows that for a fixed Δn when $x < a/2$, the error $\Delta\alpha_2$ through determination by counter curves is less than the error $\Delta\alpha'_2$ through determination by a simple curve. For $x = a/2$, the reverse relation is true, i.e. $\Delta\alpha_2 > \Delta\alpha'_2$. In Figure 12, the curve $\Delta\alpha'_2/\Delta n$ which was computed from (60) is shown by the dotted line. From a study of this curve it is clear that $\Delta\alpha'_2/\Delta n$ changes rapidly with distance in the region of comparatively small values of x; for $x > 150$-200 meters the value of changes slowly with distance. Therefore, curve (58) with a known degree of accuracy can be approximated as a straight line with angular coefficient equal to $(\alpha_2 + \overline{\Delta\alpha'})$, where $\overline{\Delta\alpha'}$ is some mean error.

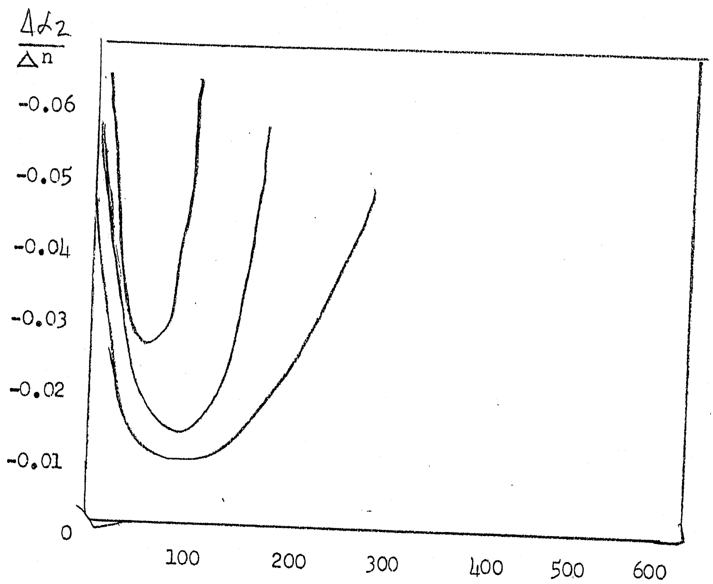

Figure 12. Curve relationships of the ratio of error $\Delta\alpha_2$ in determining the coefficient of absorption α_2 to the error Δn in determining n from distance x; unbroken lines are curves of errors for determination of α_2 by counter amplitude curves; the parameter of the family is the distance a between epicentres; the dotted line is is the curve of errors when α_2 is determined from a single amplitude curve.

A comparison of curve (60) with the curves (57) shows that the errors in determining   by two methods -- by two counter curves and by one amplitude curve -- are essentially different, especially for sections of curves which lie near epicentres. These differences in the value of α_2, determined by the two methods, in some cases may be used for detection of error in determining the value of n.

The Necessity of Accuracy in Determining n. The results of the calculations which were introduced in the last paragraph show that accuracy in determining the coefficient of absorption α_2 essentially depends on the accuracy with which the index n is determined. Assigning a limit to the allowable values of error $\Delta\alpha_2$, it is possible from the graph (Figure 12) to determine the limit of the allowable value of error Δn_{lim}; the allowable error $\Delta\alpha_2$ essentially depends on the value of α_2 in the rocks investigated. As was shown, the results of processing the experimental data obtained in 1946-1950 in the Geophysical Institute of the Academy of Science USSR by the correlation method of refracted waves, the values of the coefficient of absorption in the frequency range f = 50 -100 megacycles for sedimentary rocks equals 0.02-0.05 m^{-1} and for some crystalline and metamorphic rocks α_2 = 0.002-0.008 m^{-1}. In the case of sedimentary rocks one can assume that the allowable value of $\Delta\alpha_2$ = 0.005 m^{-1} which constitutes from 10 to 25 percent of the measured value. In the case of crystalline and metamorphic rocks the allowable error of $\Delta\alpha_2$ ought not exceed 0.001 m^{-1}, which constitutes from 10 to 50 percent of the measured value. In Table 1 by way of illustration are presented the limits of allowable values of Δn_{lim}, computed for some values of a and x and for fixed $\Delta\alpha_2$ = = 0.001 m^{-1} and $\Delta\alpha_2$ = 0.005 m^{-1}. For computations it is assumed that the determination of α_2 is derived from a system of two counter curves.

Table 1

a, in meters	Interval of the values of x, in meters	Mean value $-\Delta\alpha_2/\Delta n$, m^{-1}	Δn_{lim} $\Delta\alpha_2 = \pm 0.001\ m^{-1}$	Δn_{lim} $\Delta\alpha_2 = \pm 0.005\ m^{-1}$
200	50-150	0.011	∓ 0.09	∓ 0.45
400	75-325	0.006	∓ 0.17	∓ 0.83
600	100-500	0.004	∓ 0.25	∓ 1.25

The data of Table 1 shows that for small allowable values of $\Delta\alpha_2$ and for small distances a and x great accuracy in the determination of n is necessary. Since such accuracy in the determination of n cannot always be obtained, then for increased accuracy in the determination of α_2, it is necessary to increase the distance a between counter epicentres. If the determination of α_2 is derived from a single amplitude curve, then, as pointed out earlier, it is necessary to use sections of the curve which are sufficiently removed from the epicentre.

The indicated conditions are easily satisfied by the determination of coefficients of absorption in the case of sufficiently deep separation boundaries. For a study of separation boundaries which lie at small depths, the distance between epicentres and the intervals of wave trackability are usually small. However, in the case of small depth the requirements for accuracy in the determination of n must be especially high.

7. Examples of Determinations of the Index of Divergence n from Experimental Data

Seismogeologic Construction of the Medium. For determining the index of divergence n recordings of refracted waves are used which were obtained under the following seismogeologic conditions.

The refracting boundaries are the surface of crystalline rocks which in single zones of observation are described as granite, but in others -- as almost vertically stratified metamorphic slates. The border velocity in these rocks is different in different zones of observation and it varies in the interval from 4000 to 6000 meters per second. For determining n, recordings are used which were obtained in the parts where the refracting surface near the horizontal and border velocity is constant. The depth of stratification of this surface in different parts is from 20 to 120 meters. The covering media are quarternary argillaceous soils and clay; the mean velocity $\overline{V}$ in these rocks from the surface of the earth to the surface of crystalline rocks is practically constant for each of the sections where the determinations of n were derived, and equalled from 400 to 1200 meters per second. Substituting a stratified covering medium for a single medium of constant mean velocity $\overline{V}$, it is schematically possible to assume that all determinations of n were made for the case of a similar layer which is situated in a similar half-range.

Regulation of Sensitivity Apparatus. Recordings were obtained in 24-channel apparatus having a maximum frequency characteristic at $f = 70$ megacycles. The seismic channels were calibrated for sensitivity. This was done since on the potentiometers which regulate the sensitivity amplifier an even regulator was substituted for a graduated one. With this in mind the practical mastic potentiometers were substituted for potentiometers with sets of calibrated wire resistances. For obtaining recordings the potentiometer slides in all amplifiers were uniform; this being the case the differences in sensitivity of different channels did not exceed ± 7 percent. In such a manner the sensitivity regulators may almost directly, by the recordings obtained and the structure of amplitude curves, without any additional calculation ascertain the nature of the damping waves in respect to distance. Small differences in channel sensitivity and also differences in amplitude curves due to the condition of seismograph equipment were excluded from the qualitative calculations of n and α_2, since while recording different epicentres the sensitivity of the channels and seismograph equipment remained unchanged.

The Characteristics of Recordings used for Determining n. Refracted waves for which determinations of n were derived, were registered by recordings as to quality at first. Recordings which were obtained at a distance x were used for determination in the majority of cases in which twice the depth of the separation boundary was exceeded. In this case, in conformance with the results of theoretical works of the author (see Paragraph 1), index n is a constant. For a selection of recordings for determining n the following conditions were maintained.

1. Waves, characteristic of the surface of crystalline rocks, in recordings which were used did not interfere with other waves, for example, with waves characteristic of the separation boundary in a covering, sedimentary series; at the beginning of interference, as is well known, wave amplitudes may be considerably distorted.

2. In the recordings which were used shifts (relays) (connected to the beginning of the vertical separation boundaries in a series of crystalline rocks) of primary waves are absent; such wave shifts usually are accompanied by the occurrence of interference which distorts considerably the path of the amplitude graphs.

3. Predominant frequencies of waves were kept constant along the line of observation and equal for all epicentres.

4. Overlapping hodographs of waves, which were formed according to recordings which were used, were strictly parallel, indicating the absence, at least, of a considerable wave penetration in the refracting layer.

Examples of recordings obtained from one and the same region by two overlapping epicentres, shown in Figure 13, and in Figure 14, is shown the system of overlapping and counter hodographs, constructed according to these recordings and seismic profiles.

1840
1830
1820
1810
1800
1790
1780
1770
1760
1750
1740
1730
1720
1710
1700
1690
1680
1670
1660
1650
1640
1630
1620
1610

GR 187

GV GK 1380

Q = 200 g

h = 10 m

GK 1610-1840

0 — 0.1 — 0.3 — 0.4

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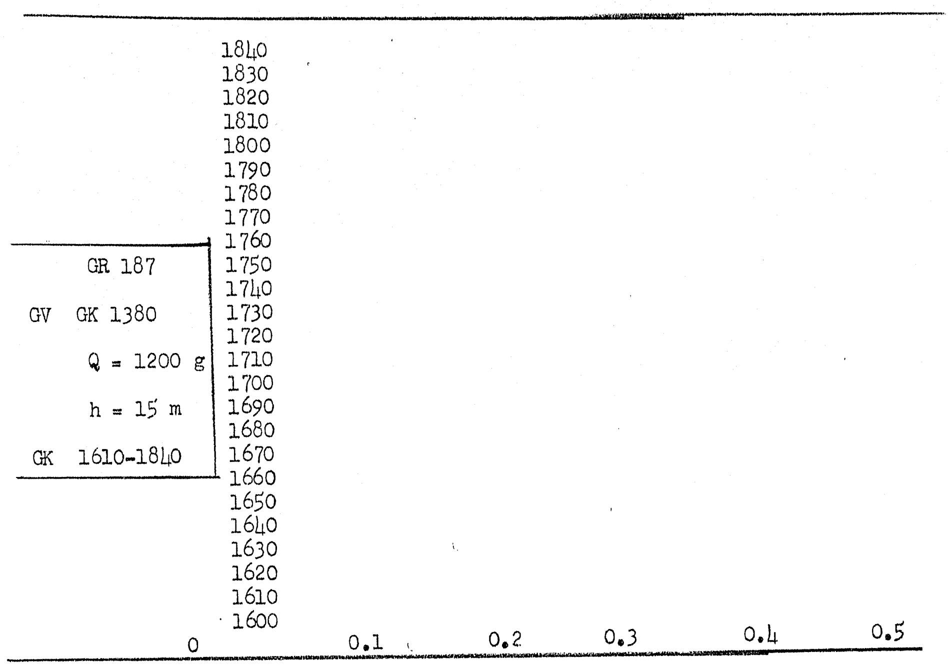

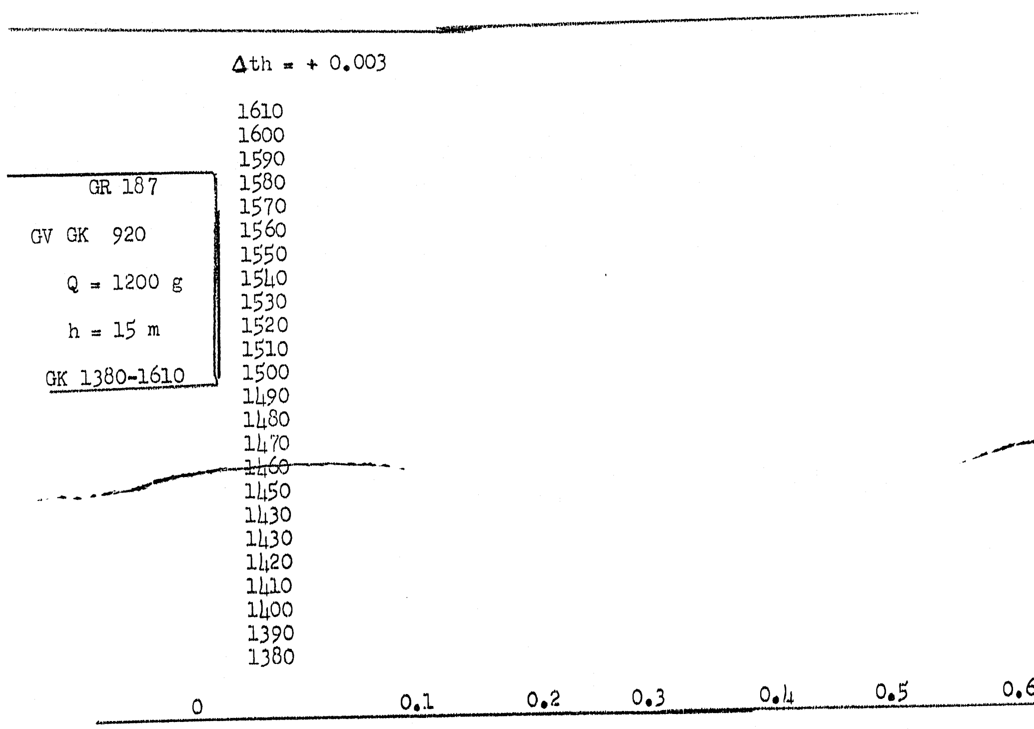

Figure 13. Seismograms obtained from the same region by two overlapping systems: a -- seismogram obtained at the epicentre GK 1380, b, c -- seismograms obtained at explosion point GK 920.

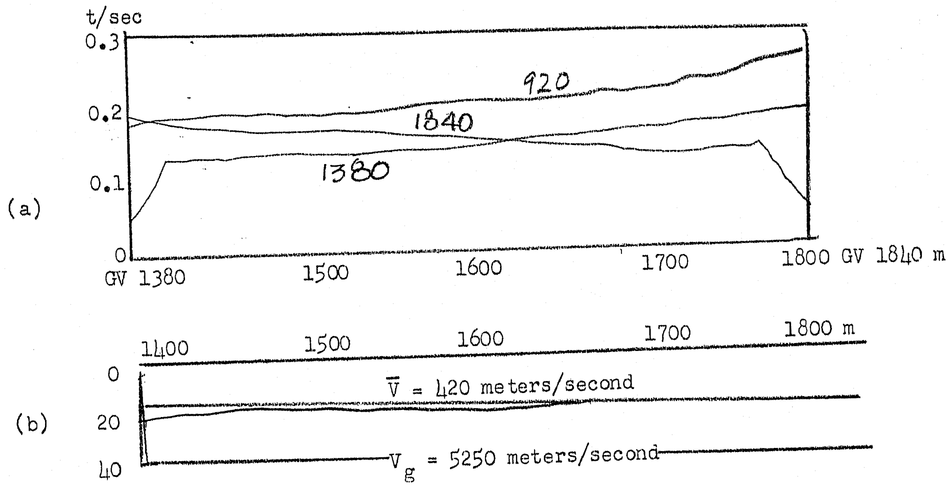

Figure 14. a -- system of overlapping and counter hodographs,

b -- seismic profile

The selection of all experimental data for the determination of n and the following treatment of amplitudes were carried out by G. H. Pariyskiy.

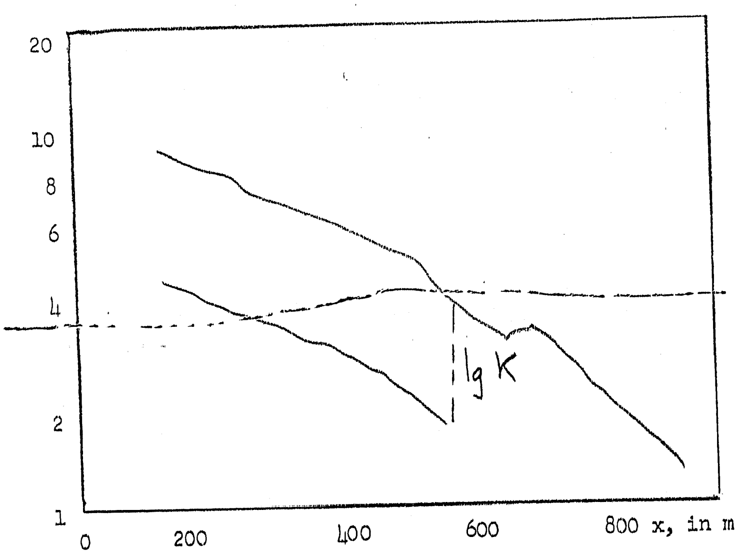

Figure 15. Construction of amplitude curves

The Variation of Amplitudes and Construction of Amplitude Curves. According to recordings, the amplitude of the primary phase of waves varies. The reading was taken between the position of the first maximum and first minimum wave (Figure 13). These variations were taken along all profiles and the variable wave amplitudes are plotted on a semilogarithmic grid (x, ln A). For two adjacent locations of seismographs the wave amplitude in the correlation channel for the most part is different. In this case the graph of amplitudes is presented in the form of two separate segments of fracturing which are staggered with respect to each other as per the value of log k (Figure 15), where k is the relation of wave amplitudes to channel correlation. In order to obtain a continuous graph of the variation of amplitudes along all intervals under investigation, it is sufficient to remove graphically one segment of the fracturing on the ordinate axis at the value of log k. If in determining n data is used which has been obtained through a series of seismograph stands then, using the indicated graphic method of parallel transfer of amplitude curve segments, which are characterized by different seismographic apparata, it is possible to obtain a continuous graph of amplitudes along all profiles.

An example of observed overlapping and counter amplitude curves is shown in Figure 16. These curves are constructed according to the same seismogram by which the hodograph shown in Figure 14 was constructed.

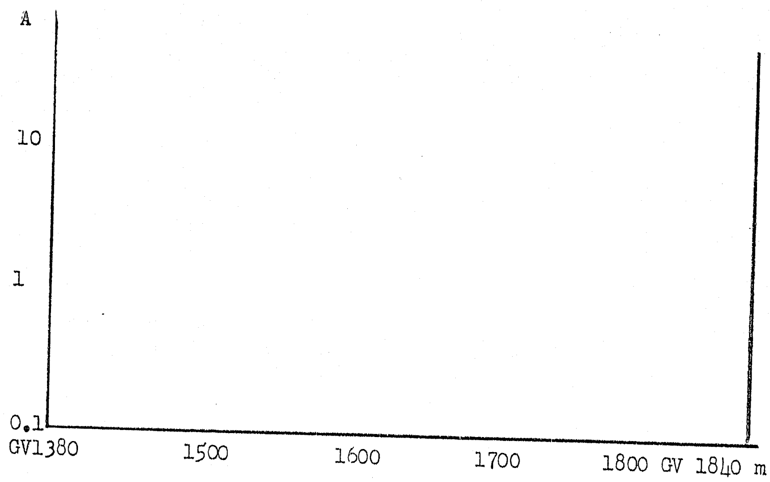

Figure 16. System of observed overlapping and counter amplitude curves

The Determination of n. The determination of n is affected by the method of overlapping curves. Linked to this is the fact that in all regions where these determinations were carried out, the refracting boundary was practically horizontal, the distance x was read along the horizontal line of observation. The distance ℓ between epicentres was equal to 460 meters. In Figure 17 examples are shown of determining n by overlapping amplitude curves, obtained in different regions of observation. The points which approximated line 1 (Figure 17), are derived from the amplitude curves shown in Figure 16. Thus, the seismograph of Figure 13, the hodograph and profile of Figure 14, the amplitude curves of Figure 16, and Graph 1 of Figure 17 give a representation of the nature of fundamental seismic materials and of seismogeologic conditions for which the value of index n = 1.34 was obtained. Data on seismogeologic conditions according to which there is also given the values of the prevailing frequency f, the ration of H/λ (depths H of stratification of the

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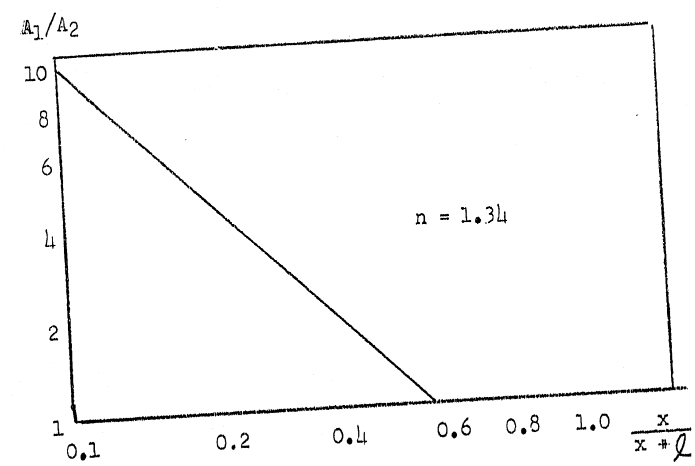
Table 2

No in sequence	Depth H, in meters	Rock	Mean velocity $\bar{V}$ in meters/second	Boundary velocity V_g in meters/second	$\frac{V}{V_g}$	Predominant frequency f in megacycles	Length of incidence waves	H / —	Distance from nearer epicentre	$\frac{x}{2H}$	n
1	20	Granite	400	5250	0.08	60	6.7	3.0	50-310	1.2-3.9	1.34
2	40	Granite	700	5800	0.12	55	12.7	3.1	160-390	2.0-8.6	1.87
3	40	Granite	700	5700	0.12	60	11.7	3.4	70-460	0.9-5.7	1.78
4	120	Slates	1000	5000	0.20	40	25.0	8.0	300-690	1.25-2.87	2.60

refracting boundary at length λ of incident waves), of distance x of theobservation points to the closer epicentre, the ratio of x/2H and the results of the determination of n.

As is seen from the examples shown in Figure 17 there is some scattering of observation points, but even from this it is possible, with a known degree of closeness, to approximate a straight line. Through single determinations, as, for example, in the case of Graph 1, a scattering of points is observed chiefly for small values of $\frac{x}{x + \ell}$, i.e. for distances of x close to an epicentre; this may be connected with the beginning of penetration. For other determinations, as, for example, in the case of Graphs 2 and 3, there is observed a scattering of points for the larger values of $\frac{x}{x + \ell}$ i.e. for the greatest distances x from an epicentre; it is possible that this indicates some increase in the index n with distance.



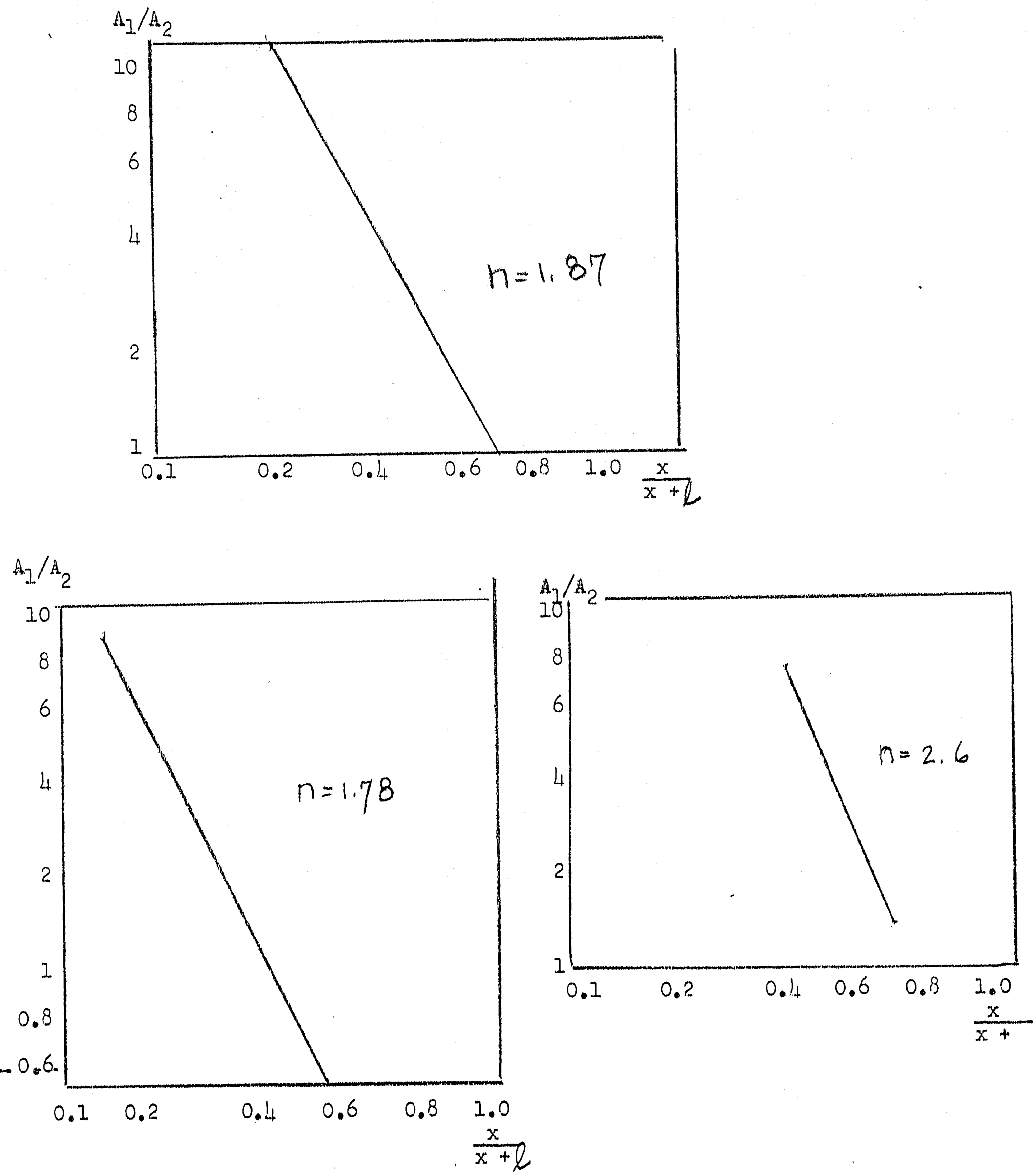


Figure 17. Graphs for determining the index n from observed data

The adduced value of n, as also the results of a series of other determinations which were made for similar seismogeologic conditions, shows that the value of n is essentially constrained to the interval 1.5-2.0. In some cases, as, for example, in the case of Graph 1, Figure 17, values of n < 1.5 were obtained and in individual cases (Graph 4, Figure 17) values of n > 2.0 were obtained. The results of treating experimental data indicate that the values of n in a great number of cases is relatively near these values which were obtained on the basis of a theoretical solution of the dynamic problem for refracted waves [8, 9, 11, 13]. Along with this, differences in the values of n are observed which were obtained as a result of a series of determinations which cannot be explained by a scattering of points. This shows that in actual media, the relation of the value of n to seismogeological conditions -- to the depth of stratification of a refracting layer, to its capacity, to the correlation of velocity in the covering medium and in the refracting layer -- is still not completely investigated. This research must be carried out on the basis of data analysis of numerous determinations of n for different cases of medium construction.

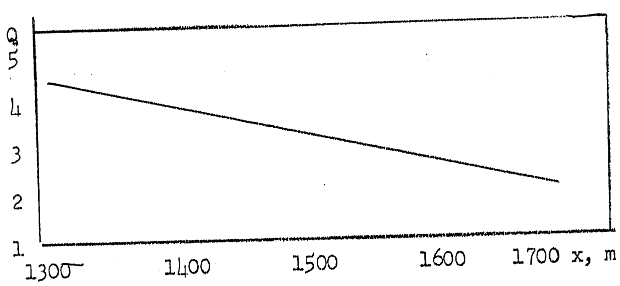


Figure 18. Graph for determining the coefficient of absorption by the system of counter amplitude curves shown in Figure 16.

To determine n, as already pointed out, except for an independent value, it is necessary to determine the coefficient of absorption α_2. The obtained values of α_2 in some cases may be used as an accuracy control for determining n. In particular, negative values of α_2 show that the value obtained for n is exaggerated. In Figure 18, in view of the example which is graphically illustrated, there was obtained a difference method for the determination of α_2 by the counter amplitude curves of Figure 16. For the determination the value of n = 1.34 was accepted which is derived from overlapping amplitude curves of the same system (Figure 17, Graph 1). As is seen from an investigation of Figure 18, the observed points which are relatively reliable may be approximated by a straight line. The coefficient of absorption, determined by the slope of this line, equals 0.0026 m^{-1}. The space decrement of wave absorption is presented as the well known formula

$$\vartheta = \alpha_2 \lambda_2$$

where λ_2 is the wavelength in the refracting layer.

When the predominant frequency f = 55 megacycles and the border velocity V_g = 5250 meters per second, the wavelength λ_2 = 95 meters. In this case ϑ = 0.25. The values obtained of α_2 and ϑ are very likely for crystalline rocks.

Conclusion

This paper has been devoted to the problem of the quantitative treatment of amplitudes of refracted waves which were recorded during work on the correlation method of refracted waves. In this work methods have been suggested for determining the exponent n = const of the divergence function for refracted waves according to observed overlapping and counter amplitude curves. The method of overlapping amplitude curves is more accurate and easier for practical application. It is known that this method is utilized as in the case of a constant coefficient of absorption α_2 in the refracting layer, as in the case when α_2 varies along the refracting boundary.

The problem was investigated of the application of the method suggested for determining n in the case when the values of n and α_2 are functions of distance from the oscillation source, and it was shown that in this case the effective value n_{eff} which is determined from observed data, may differ greatly from the true value of n.

It has been shown that the accuracy of determining the coefficient of absorption in the refracting layer, essentially depends on the accuracy of the determination of the index of divergence n.

In the last paragraph examples were presented of the determination of the index of divergence n according to experimental data.

The calculation of n according to experimental data by the method suggested showed that the values of n generally are equal to 1.5-2.0, but in some cases larger as well as smaller values of n are obtained. Subsequently, on the basis of the results of mass determinations of n, it is necessary to establish the relation of the index n to stratification depth and capacity h of the refracting layer, to the ratio of velocity $\overline{V}/V_g$ in the covering medium and in the refracting layer and to the predominant frequencies f of refracted waves. Particular attention must be paid to the study of the relation of the value of n to distance x and to improving the methods for determining n in the case of the variable $n = n(x)$.

The determination of the index n offers great interest for the study of the problem of the physics of the propagation of refracted waves in different geological media. Simultaneously with this the determination of the value of n is basically necessary also for an investigation of refracting properties of different media;

whereas for determining the coefficient of absorption α_2 it is necessary to know the value of n. The determination of n must proceed along with the determination of α_2. For this it is necessary to obtain systems of overlapping and counter amplitude curves. The combination of two parameters n and α_2, which are determined according to this system, denotes the degree of stratification of refracted waves with distance in different media. The employment during the survey of data on the damping of refracted waves with distance, together with the utilization of other dynamic properties of waves and time of run to permit a fuller study of the seismogeological structure of the medium and, undoubtedly, to broaden the exploring possibilities of the correlation method of refracted waves.

In conclusion, let us note that the described method for determining the exponent of the divergence function n for refracted waves is applicable to straight line and surface waves. In a solid medium the functions of divergence for these waves may essentially differ from the asymptotic formulas which were obtained from the equation of the theory of elasticity for an ideally elastic medium.

In particular, it is extremely interesting to ascertain, on the basis of an analysis of the values of n, whether the waves registered at a close distance to an epicentre are direct or whether they are refracted waves related to a thin layer with an increased velocity, which are often found in the similar appearing media from a geological point of view.

The described method for determining the index n is also expedient to use for treatment of amplitude curves obtained through observation by the method of stationary oscillations. In this case, with great care, a study of the values of n may assist in determining types of waves induced at the source of sinusoidal oscillations of fixed frequency.

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